Total No. of Questions-30
Total No. of Pages-4

Roll No. : | | | | | | |

Half Yearly Examination 2019-20

Class : 12

BSE-860

Subject : Mathematics

Time : 3.15 Hours **M.M. : 70**

Note : (i) All Questions are Compulsory.

(ii) Write the answer to each question in the given answer book only.

(iii) There are internal choice bewtween Q. No. 16, 24, 28 and 30 you have to attempt only one if the alternatives in these questions.

(iv)

Section	*Q. No.*	*Marks*
A	*1-15*	*1*
B	*16-25*	*3*
C	*26-30*	*5*

PART-A

1. If $\tan^{-1}(1)+\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)=\sin^{-1}x$ then find the value of x.

2. Find the devivative of $\sin x^{\circ}$.

3. Find the value of the integrates $\int \cot^2 x\,dx$.

4. Find the value of the following integrate $\int_0^{\infty}\frac{\sin\left(\tan^{-1}x\right)}{1+x^2}dx$.

5. Integrate with respect to x :

$$\frac{1}{\sqrt{9-25x^2}}$$

6. Integrate the following with respect to x :

$$\frac{e^{\sqrt{x}}.\operatorname{cose}^{\sqrt{x}}}{\sqrt{x}}$$

7. Find the area enclosed by curve $y=2\sqrt{x}$ and $x = 0$, $x = 1$.

P.T.O.

8. Solve the following Diffective equations :

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

9. Find the order and degree of the following differential equation :

$$y = x\frac{dy}{dx} + \frac{a^2}{\frac{dy}{dx}}$$

10. Find the projection of the vector $4\hat{i} - 2\hat{j} + \hat{k}$ on the vector $3\hat{i} + 6\hat{j} - 2\hat{k}$.

11. Find the perpendicular unit vector of the vectors $\hat{i} - 2\hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} - 3\hat{k}$.

12. Find the equation of the line parellel to the vector $2\hat{i} - \hat{j} + 3\hat{k}$ and pass through the point (5, –2, 4).

13. If a vector makes angles of α, β and γ with the OX, OY and OZ axis respectively, then prove that :

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

14. Find the solution of differential equation :

$$e^{-x+y}\cdot\frac{dy}{dx} = 1$$

15. If the coordinates of P and Q are (3, 4) and (12, 9) respectively, then find $\angle POQ$, where O is origin.

PART-B

16. If f R $\to$ R such that $f(x) = ax + b$, $a \neq 0$ then prove that f is a bijection. Also find f^{-1}.

OR

Find f^{-1}, Such that f: A $\to$ B, A = {0, – 1, – 3, 2}, B = {–9, – 3, 0, 6} and $f(x) = 3x$ (if exists)

17. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$ then prove that $xy + yz + zx = 1$.

18. If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$ then prove that $A^n=\begin{bmatrix}\cos n\alpha & \sin n\alpha\\ -\sin n\alpha & \cos n\alpha\end{bmatrix}$.

19. Find the value of determinant $\begin{vmatrix}\frac{1}{a} & a^2 & bc\\ \frac{1}{b} & b^2 & ca\\ \frac{1}{c} & c^2 & ab\end{vmatrix}$.

20. Show that $A=\begin{bmatrix}2 & -3\\ 3 & 4\end{bmatrix}$ satisfies the equation $A^2 - 6A + 17 = 0$.

Thus find A^{-1}.

21. Examine for diffentiability of the function $f(x) = |x - 1| + |x|$ at $x = 0, 1$.

22. If $y = a \cos nx + b \sin nx$ then prove that $\frac{d^2y}{dx^2}+n^2y=0$.

23. Integrate of $\frac{1}{\sqrt{5x-6-x^2}}$ with respect to x.

24. Find the area enclosed by ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$, and lie above the x axis.

OR

Find the area of the region in the first quadrant enclosed by the para bola $y^2 = 4ax$ and line $y = x$.

Solve the following differential equation $(x+y)=\sin^{-1}\left(\frac{dy}{dx}\right)$.

P.T.O.

PART-C

26. A ladder 13m long leans against a wall. The foot of the ladder is pulled along the ground away from the wall, at the vate of 1.5 m/sec. How fast is the angle θ between the ladder and ground is changing when the foot of the ladder is 12 m away from the wall.

27. For curve $y = \sin^2 x$, find the equation of normal at $\left(\frac{\pi}{3}, \frac{3}{4}\right)$.

28. Show that for any vector $\vec{a}$, $\left|\vec{a} \times \hat{i}\right|^2 + \left|\vec{a} \times \hat{j}\right|^2 + \left|\vec{a} \times \hat{k}\right|^2 = 2\left|\vec{a}\right|^2$.

OR

If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + 3\hat{j}$ such that $\vec{a} + \lambda\vec{b}$ is perpendicular on $\vec{c}$, then find the value of λ.

29. Solve the following linear programming problems by graphical method :

Minimize $z = 3x + 5y$

Subject to the constraints

$$x + 3y \geq 3$$
$$x + y \geq 2$$

and

$$x \geq 0$$
$$y \geq 0$$

30. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersects. Find their point of inter section.

OR

Determine the value of m if line $\vec{r} = \left(\hat{i} - 2\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$ is parallel to the plane $\vec{r}.\left(3\hat{i} - 2\hat{j} + m\hat{k}\right) = 3$.

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